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The Optimal Level of Reserves for Low-Income Countries: Self-Insurance against External Shocks

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This paper develops an analytical framework that helps to quantify the optimal level of international reserves for a small open economy with limited access to foreign capital and subject to natural disasters or terms-of-trade shocks. International reserves allow the country to relieve balance of payments pressures caused by external shocks and to avoid large fluctuations in imports. The paper calibrates the model to two regions—the Caribbean and the Sahel region in sub-Saharan Africa—and assesses the sensitivity of the results. The conclusion is that popular rules of thumb, such as maintaining reserves equivalent to three months of imports, only give imprecise benchmarks. [JEL F30, F31, F32]

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What is the optimal amount of international reserves for countries with limited access to foreign capital? While the recent buildup in international reserves in Asia spawned a renewed interest in the appropriate level of reserves for emerging market economies, less developed countries have largely been ignored by the literature. As a result, policymakers rely on personal judgment or rules of thumb such as maintaining reserves equivalent to three months of imports to evaluate a country's needs.

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This paper develops an analytical framework that helps to quantify the level of reserves that can be rationalized in terms of insurance against large external shocks, such as natural disasters or terms-of-trade shocks. By calibrating the model, the paper estimates the optimal amount of international reserves for two groups of countries subject to different natural disasters: the Caribbean, subject to hurricanes, and the Sahel region in sub-Saharan Africa, subject to drought.[1]

Although there are a number of reasons to accumulate international reserves, many low- to middle-income countries have weakly diversified economies that are very vulnerable to natural disasters or terms-of-trade shocks. Indeed, less developed countries often rely on international trade to import large quantities of goods of prime necessity (such as food) and on a single export sector to generate most of the foreign-exchange inflows. In addition, and unlike middle- to high-income countries, they do not have fast access to private foreign capital and must rely on international and bilateral donors to meet emergency financing needs. Their reliance on such flows, however, has considerable disadvantages. It can take time before donor resources are committed and disbursed, and there may be competition for donor resources from other countries with relief needs at the same time. In this context, international reserves can play a critical role by allowing rapid access to foreign exchange to avoid large imports fluctuations due to balance of payment constraints.

This paper presents a simple model that helps to quantify the level of reserves that can be rationalized in terms of insurance against large external shocks. The model looks at the intertemporal optimization problem of a small open economy that can hold costly foreign reserves to smooth import fluctuations in the face of large external disturbances. Because of the balance of payment constraint, a country can only buy imports if it receives enough foreign exchange inflow. By suddenly disrupting the normal inflow of foreign exchange, a natural disaster or a terms-of-trade shock may prevent a country from importing the desired level of foreign goods, resulting in a welfare loss. With an appropriate amount of international reserves, a country can minimize the negative impact of such shocks. Under a few assumptions, one can simplify the problem and derive a closed-form solution for the optimal reserves-to-import ratio that depends on the frequency and duration of shocks, the economic damage, the economy's characteristics, and the opportunity cost of holding reserves. Using data on natural disasters and terms-of-trade shocks since 1960, this paper then calibrates and numerically solves the fully fledged model to estimate the optimal amount of international reserves for the Caribbean and the Sahel countries.

The popular rule of thumb of maintaining reserves equivalent to three months of imports gives only an imprecise benchmark, as small changes in key parameters such as the shock's persistence, the size of the export sector,

[1]See Table A1 for a list of countries for each region.

or the degree of risk aversion can have large consequences on the optimal reserves level. Although an average Caribbean country needs only about a month and half of imports, an average Sahel country needs over four months. Hence, rules of thumb can only give an imprecise benchmark, and a careful study of each country's characteristics is necessary to evaluate its needs.

It is also important to stress that these estimates constitute only a lower bound on the appropriate level of reserves, as countries need to accumulate reserves to achieve other objectives beyond self-insurance against natural disasters and terms-of-trade shocks. Providing liquidity when needed or limiting exchange rate volatility (or maintaining a fixed peg) are perfectly good reasons that this framework will brush aside.[2]

The normative literature on the optimal level of international reserves goes back to the 1960s with Heller (1966), whose main insights were later formalized in a Baumol-Tobin inventory framework with fixed costs of adjusting reserves, and in which the stock of reserves is being depleted by a stochastic current account deficit (for a review, see, Frenkel and Jovanovic, 1981; Flood and Marion, 2002). More recently, the buildup in international reserves in Asia spawned renewed interest in the optimal level of reserves for emerging market countries prone to sudden-stops in capital inflows. Jeanne and Ranciere (2008), Aizenman and Lee (2005), Caballero and Panageas (2007), and Durdu, Mendoza, and Terrones (forthcoming) present models of optimal international reserves, in which countries aim to self-insure against sudden stops in capital inflows.[3] However, less developed countries have largely been ignored by the literature given their limited access to foreign private capital.[4] The framework presented here borrows from the vast literature on precautionary savings by modeling a low-income country as a representative agent with no access to the (international) capital market.[5] In the face of income uncertainty—represented by the occurrence of natural disasters—the agent's only option is to self-insure by managing a stock of riskless assets to buffer its consumption against adverse shocks. However, unlike standard models of precautionary savings, goods are not storable, and a precautionary savings motive emerges because of the balance of payment constraint as the country accumulates international reserves to avoid low consumption levels of imported goods. The model is closest to Jeanne and

[2]A country may also accumulate reserves if it pursues an export-led growth by artificially maintaining an undervalued exchange rate.

[3]See also Alfaro and Kanczuk (2009), who study the joint decision of holding reserves *and* sovereign debt, whereas most of the aforementioned literature tends to take the level of international debt as given.

[4]An exception is Aslam and Kim (2007), who study the optimal amount of precautionary savings in the face of volatile aid flows.

[5]For studies of precautionary savings, see Zeldes (1989); Caballero (1990); Kimball (1993); Carroll and Kimball (1996); and Hugget and Ospina (2001).

Ranciere (2008) but with two main differences. It is tailored to low-income countries with no access to capital markets, and it is explicitly dynamic; that is, a country can face more than one external shock over time.

I. Natural Disasters in the Caribbean and the Sahel

Developing countries are vulnerable to terms-of-trade shocks because they typically rely on a concentrated export sector to generate most of their foreign-exchange inflows. However, for many of these countries, the problem is exacerbated by geographical location. Figures 1 and 2 respectively display the number of people affected by natural disasters over 1963–2007 for the Caribbean and the Sahel.[6] Although the former is regularly hit by hurricanes, the latter suffers frequently from droughts. By disrupting the export sector and the normal inflow of foreign exchange, natural disasters can trigger balance of payments pressures in the same way that terms-of-trade shocks do. This section estimates the economic impact of natural disasters in each region, and calculates the average behavior of real output growth, real export growth, real import growth, and the change in the nominal exchange rate in a five-year event window centered on a shock.[7]

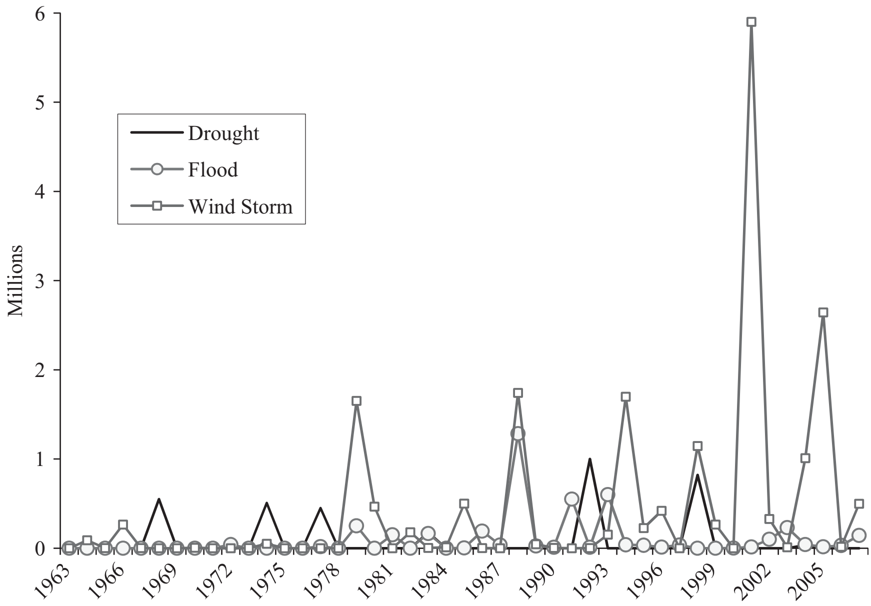
Hurricanes in the Caribbean

In September 2004, a Category 3 hurricane hit the island of Grenada and caused estimated damage of over US$800 million—or twice Grenada's GDP. Just as it required additional resources to finance relief, cleanup and emergency rehabilitations, the island experienced a dramatic decline in revenues and export earnings. Tourism and agriculture, the two major sources of foreign exchange earnings, were hit hard. Most tourism facilities could not reopen for the next six months, while the nutmeg crop, the principal export commodity, was largely destroyed. The government sought donor assistance, but despite over $150 million in pledges, only $12 million was available to address the immediate liquidity needs. Instead of focusing on recovery and reconstruction, the government was distracted by the need to

[6]Data on natural disasters are drawn from the Emergency Events Database (EM-DAT) published by the Centre for Research on the Epidemiology of Disasters (CRED) (www.em-dat.net).

[7]A good proxy for a hurricane's strength is the destruction of capital that it generates. Hurricanes are defined here as "major" when the estimated damage amounts (reported in EMDAT) represent more than 10 percent of a country's GDP. Droughts do not generate direct damage but rather hurt the population by disrupting production and/or triggering episodes of famine. Hence, a drought is classified here as "major" when either 10 percent of the population is affected *or* when at least five droughts occurred during the year. These thresholds allow for capturing most natural disasters with major consequences while ignoring smaller and more localized disasters that only had a minor impact on production and exports. With these threshold values, one obtains 30 observations (that is, disasters) for the Caribbean and 41 for the Sahel. All results are robust to alternative thresholds.

Figure 1. Total Number of People Affected by Type of Natural Disaster in the Caribbean



Note: Data on natural disasters are drawn from the Emergency Events Database (EM-DAT) published by the Centre for Research on the Epidemiology of Disasters (www.em-dat.net). People are considered "affected" when injured, homeless, displaced, evacuated, or requiring immediate assistance during a period of emergency.

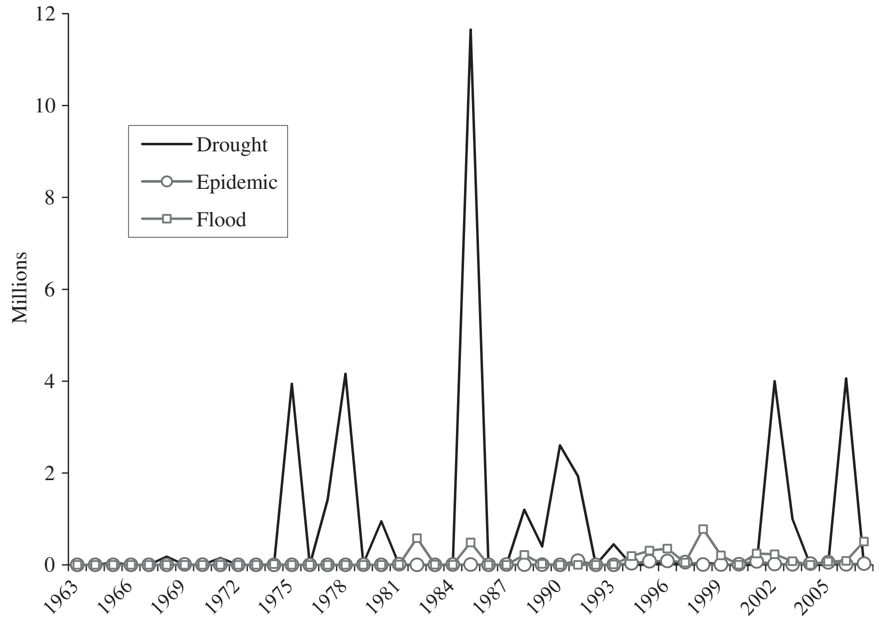
finance the emerging resource gap. This led to delays in the recovery and reconstruction periods.[8]

This episode illustrates the fate of many island nations from the Caribbean that suffer regularly from hurricanes, with enormous costs to the stock of capital and disruptions to the productive apparels. Figure 3 presents the average economic impact of major hurricanes in the Caribbean with one standard-deviation error band. On average, a major hurricane hits a Caribbean country every 20 years, that is, with a probability $\pi^{nd} = 0.95$ each year.[9] Output growth falls by 3 percentage points while exports fall by 5 percentage points. Despite the shortfall in foreign-exchange earnings, imports

[8]Despite the fact that many Caribbean islands are not, strictly speaking, low-income countries and theoretically have access to international capital markets, their high debt level limits their ability to access credit in the aftermath of a disaster. In addition, access to catastrophe insurance is limited due to the high transaction costs resulting from the relatively small amount of business island nations bring to these markets.

[9]Because natural disasters data are only available at an annual frequency, this data set cannot be used to estimate the disasters duration.

Figure 2. Total Number of People Affected by Type of Natural Disaster in the Sahel



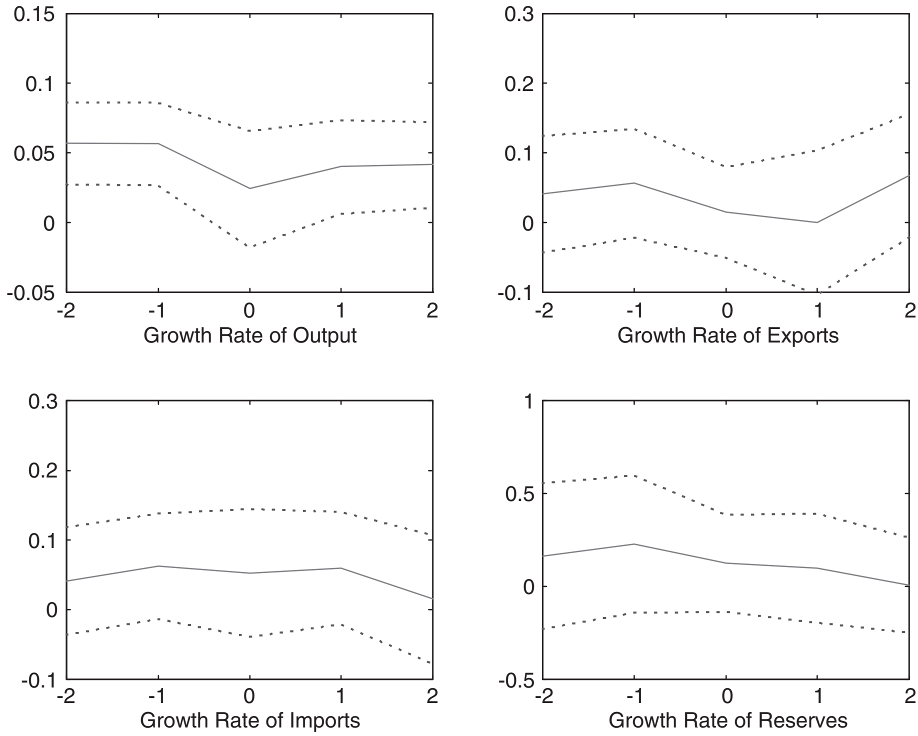
Note: Data on natural disasters are drawn from the Emergency Events Database (EM-DAT) published by the Centre for Research on the Epidemiology of Disasters (www.em-dat.net). People are considered "affected" when injured, homeless, displaced, evacuated, or requiring immediate assistance during a period of emergency.

do not decline; in fact, the country's import needs for reconstruction purposes are particularly large. As a result, import growth is relatively stable, declining by an average of only 1 percent, while the growth rate of reserves declines by an average of 10 percentage points on impact and by an additional 10 percentage points in the second year after the shock.

Droughts in the Sahel

Figure 4 presents the average economic impact of major droughts in the Sahel region with one standard-deviation error band. An average country from the Sahel faces one major drought every 12 years, that is, with a probability $\pi^{nd} = 0.92$ each year. Unlike hurricanes, droughts tend to develop over the course of several years. Although the behavior of real economic variables resembles qualitatively that of Caribbean countries, there are quantitative differences. Output growth drops only marginally by 0.3 percentage points on impact before rebounding the next year, but export growth drops by 8 percentage points. Imports remain roughly constant while the growth rate of reserves falls by 16 percentage points.

Figure 3. Impact of a Major Hurricane in the Sahel

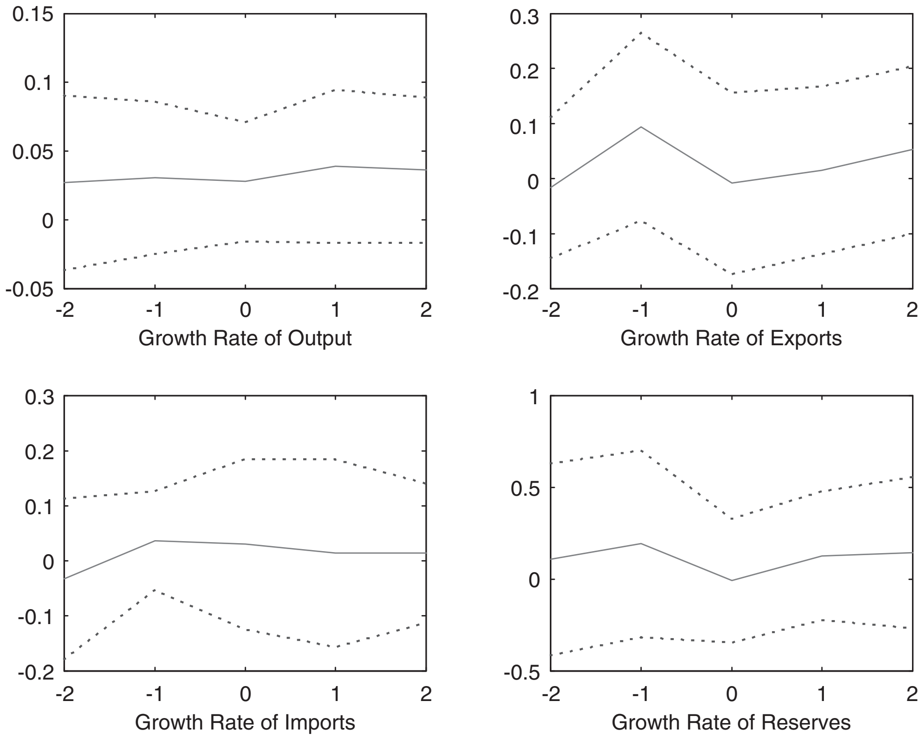


Note: The five-year event window is centered around a natural disaster occurring at time 0. A drought is considered "major" when either 10 percent of the population is affected or at least five droughts occurred during the year. Dashed lines represent the one standard-deviation error bands.

II. A Model of Optimal International Reserves

This section presents and calibrates a simple model of a small open economy that cannot access international capital markets but can hold costly international reserves to smooth consumption fluctuations in the face of large terms-of-trade shocks or large disturbances to exports. The optimization problem of this small open economy is analogous to the optimization problem of a single individual in the heterogenous-agents models of precautionary savings (for example, Hugget, 1993; Aiyagari, 1994). As in those models, Constant Relative Risk Aversion (CRRA) utility with incomplete markets implies that the marginal utility of consumption goes to zero from above, making the economy "extremely averse" to a savings plan that would leave it exposed to the risk of "very low" consumption at any date and state of nature. However, unlike standard models of precautionary savings, goods are not storable, and a precautionary savings motive emerges because of the balance of payment constraint. The model economy consumes

Figure 4. Impact of a Major Drought in the Sahel



Note: The five-year event window is centered around a natural disaster occurring at time 0. A drought is considered "major" when either 10 percent of the population is affected or at least five droughts occurred during the year. Dashed lines represent the one standard-deviation error bands.

two types of goods—domestically produced or imported—but without access to international capital markets; it can only buy imports if it receives enough foreign exchange inflow. More specifically, a country can import foreign goods by (1) exporting home goods, (2) borrowing or receiving grants from abroad, or (3) using foreign exchange reserves. By suddenly disrupting exports and the normal inflow of foreign exchange, a natural disaster or a terms-of-trade shock may prevent a country from importing the desired level of foreign goods, resulting in a welfare loss. By holding an appropriate amount of international reserves, a country can minimize the negative impact of such shocks.[10] However, this self-insurance comes at a price because of the opportunity cost of accumulating low-yield securities such as U.S. government bonds.

[10]Middle- to high-income countries can address the immediate liquidity needs by borrowing abroad but this is not the case for less developed countries with no immediate access to private foreign capital.

The Model

There are two countries, "Home and Foreign." Home is a small open economy consisting of a representative agent that consumes two types of goods: home goods c_H and foreign goods c_F. Both goods are not storable. At each period t, the Home consumer receives an endowment Y_t of home goods that can either be consumed or exported. The economy grows at the rate g so that $Y_t = (1+g)Y_0$ and in this nonstationary economy, all variables are expressed as a share of "normal" output (that is, the output level prevailing in a nondisaster state). In order to import foreign foods, Home must pay in foreign currency so that at each date t, it must satisfy the balance of payments constraint

$$c_{F,t} \leq \varepsilon_t c^*_{F,t} - ((1+g)R_{t+1} - R_t) + Tr_t,$$

where $c_{F,t}$ is the import-to-output ratio, $c^*_{F,t}$ is the export-to-output ratio, ε_t is the terms of trade, R_t is the reserves-to-output ratio (the amount of international reserves scaled by output) and Tr_t is a generic term for foreign transfers (private remittances or official grants) and loans as a share of output.[11]

Holding low-yield reserves presents an opportunity cost to Home modeled as a payment rR_t, payable in home goods. Indeed, Jeanne (2007) argues that instead of accumulating reserves in the form of, say, low-yield U.S. bonds, economies could receive a higher rate of return by investing in the domestic business sector or in the building of public infrastructure.[12] Hence, the aggregate resource constraint (rescaled with the output level) takes the form

$$c_{H,t} + \frac{1}{\varepsilon} c_{F,t} = y_t - \frac{1}{\varepsilon}(rR_t + ((1+g)R_{t+1} - R_t) - Tr_t),$$

with y_t denoting the country's endowment as a share of its income in "normal" times.

The representative agent seeks to maximize its expected utility by consuming home and foreign goods subject to the aggregate resource constraint and the balance of payment constraint. At date 0, Home's problem can be written

$$\max_{\{c_{H,t}, c_{F,t}\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_{H,t}, c_{F,t})$$
$$\text{s.t.} \begin{cases} c_{H,t} + \frac{1}{\varepsilon} c_{F,t} = y_t - \frac{1}{\varepsilon}(rR_t + (R_{t+1} - R_t) - Tr_t) \\ c_{F,t} \leq \varepsilon c^*_{F,t} - ((1+g)R_{t+1} - R_t) + Tr_t \\ R_{t+1} \geq 0 \end{cases}.$$

[11] Home takes ε_t as given as the low-income country is too small compared to the rest of the world to affect world markets and the terms of trade.

[12] Jeanne (2007) also measures the opportunity cost of reserves by using the spread between the interest rate on external debt and the return on reserves. However, it is difficult to apply this approach to low-income countries with no or little access to private capital market and whose external debt consists mostly of subsidized loans from international organizations or foreign governments.

To capture the occurrence of rare disasters and their impact on the economy, the country's endowments of home goods as well as the value of its exports follow a two-state Markov process with time-invariant transition probabilities. In a "normal" state, the representative agent receives an endowment Y^n and exports a fraction of output $c_F^{*n} = \delta$. However, with probability π^{nd}, a natural disaster hits the economy in a "normal" state and disrupts output production, exports capacities, and the terms of trade such that $Y^d = \eta_Y Y^n$, $c_F^{*d} = \eta_X c_F^{*n}$ and $\varepsilon^d = \eta_\varepsilon \varepsilon^n$ with η_Y, η_X, $\eta_\varepsilon < 1$. Once in a "disaster" state, the economy returns to its "normal" state with probability π^{dn} so that $1/\pi^{dn}$ is the expected duration of the disaster.

Because foreign donors typically decide unilaterally on the aid amount they provide to less developed countries, it is assumed that Foreign provides an exogenous and constant aid-to-GDP ratio Tr each period. Hence, in the aftermath of a disaster, Home can only cover its foreign exchange losses by using international reserves.[13] By denoting $\tilde{c}_{F,t} = c_{F,t} - Tr$ the imports that are only paid for with international reserves or the proceeds of exports, and $\mathscr{S}_{\{s=d\}}$ an indicator function equal to zero in a normal state and one in a disaster state, one can rewrite Home's problem at date t as follows:

$$V(R_t) = \max_{R_{t+1}} \left[u(c_{H,t}, \tilde{c}_{F,t}) + E_t \beta V(R_{t+1}) \right]$$

$$\text{s.t.} \begin{cases} c_{H,t} = (1-\delta) y_t - r R_t / \varepsilon_t \\ \tilde{c}_{F,t} \leq \varepsilon_t \delta_t y_t - (1+g) R_{t+1} + R_t \\ y_t = 1 - (1-\eta_Y) \mathscr{S}_{\{s_t = d\}} \\ R_{t+1} \geq 0 \end{cases}.$$

Home will choose its level of international reserves to satisfy the first-order condition $u'_{C_F,t} = \frac{\beta}{1+g} E_t \left(u'_{C_F,t+1} - \frac{r}{\varepsilon_{t+1}} u'_{C_H,t+1} \right)$. By accumulating one more unit of reserves in period t, Home gives up on consumption of foreign goods at t and of home goods at $t+1$ because of the opportunity cost of reserves, but it also enjoys a higher expected utility of foreign goods consumption at $t+1$. Higher income growth has the same effect as a higher discount rate as the representative agent would like to increase consumption and borrow in anticipation of higher future income.

In the steady-state of the "normal" state, Home reaches its optimal reserves to output ratio R^* and its consumptions of home and foreign goods are

$$\begin{cases} c_H = 1 - \delta - r R^* / \varepsilon \\ c_F = \varepsilon \delta^n - g R^* + Tr \end{cases}.$$

[13]This assumption remains valid for a time horizon of a few months after the shock. While the IMF, World Bank, and bilateral donors do provide emergency assistance for countries hit by natural disasters, the process can be lengthy, and the funds are usually not available until a few months after the shock.

An Approximated Closed-Form Solution for the Reserves-to-Imports Ratio

This subsection studies the problem analytically by considering a simpler case with the log-utility specification

$$u(c_{H,t}, c_{F,t}) = \theta \ln(c_{H,t}) + (1-\theta)\ln(c_{F,t}) \text{ with } \theta \in [0,1].$$

The country's first-order condition can be simplified by noting that $r \ll 1$, $g \ll 1$, $R \ll 1$ and $\pi^{nd} \ll 1$. Indeed, the opportunity cost of reserves will typically be smaller than 10 percent, and developing countries rarely have reserves-to-output ratio in excess of 10 percent. We also saw in Section I that disasters are extremely rare events occurring less than once every 10 years, so that π^{nd} is less than 1 percent. In that case, the Appendix shows that Home's first-order condition implies

$$R^* - (1+g)R' \approx \frac{\pi^{nd}}{\frac{1}{\delta\varepsilon}\left(\frac{1+g}{\beta} - (1-\pi^{nd})\right) + (1-\pi^{nd})\frac{r\theta/\varepsilon}{(1-\delta)(1-\theta)}} - \varepsilon\delta\eta_\varepsilon\eta_X\eta_Y,$$

with R^* the optimal reserves-to-output ratio and R' the level of reserves immediately after a disaster. Further, assuming that the agent uses almost all of its reserves at once so that $R' \ll R^*$, the resulting expression for the optimal reserves-to-import ratio is[14]

$$\frac{R^*}{c_F} \approx \varepsilon\delta\left[\frac{\pi^{nd}}{\left(\frac{1+g}{\beta} - (1-\pi^{nd})\right) + (1-\pi^{nd})\frac{\delta}{1-\delta}\frac{r\theta}{1-\theta}} - \eta_\varepsilon\eta_X\eta_Y\right] \times \frac{1}{\varepsilon\delta + Tr}. \quad (1)$$

Looking at Equation (1), one can draw a number of intuitive conclusions on the determinant of the optimal amount of reserves. A higher shock probability (that is, a higher π^{nd}) or a larger drop in the value of exports (that is, a lower η_Y, η_X or η_ε) raises the optimal reserves-to-import ratio. On the other hand, a higher opportunity cost of holding reserves (that is, a higher r), a higher discount rate (lower β) or higher growth rate (higher g) lowers the reserves-to-import ratio.[15] The share of imports covered by foreign grants or loans Tr influences the level of optimal level of reserves: a higher level of transfers (official loans or grants, and private remittances) in steady-state lowers the optimal reserves-to-import ratio as transfers or loans

[14] This assumption is clearly a restriction, but simulations show that with log-utility, it holds for country groups such as the Caribbean that face very disruptive but rare and short-lived disasters.

[15] As mentioned previously, higher income growth has the same effect on optimal reserves as a higher discount rate. Indeed, the model economy is equivalent to an economy with zero growth but with a discount rate $\tilde{\tilde{\beta}} = \frac{\beta}{1+g}$.

are not sensitive to natural disasters.[16] Finally, there is an inverse U-shape relationship between the size of the export sector δ and the optimal reserves-to-output ratio R^*. This is due to the interaction of two factors: the utility cost of accumulating reserves and the opportunity cost of maintaining a given level of reserves. Given the concavity of the utility function, higher foreign-exchange inflows (that is, a larger export sector) make reserves accumulation relatively easier, and the optimal reserves-to-import ratio increases (as captured by the first term $\varepsilon\delta$ on the right-hand side of equation (1)). However, above a certain level, the country exports and imports such a large share of its GDP that the steady state level of reserves gets large relative to GDP, and the opportunity cost of holding reserves becomes nonnegligible (that is, $\frac{\delta}{1-\delta}r$ becomes large in equation (1)).

III. Calibration and Numerical Solution

This section calibrates the model, calculates the optimal reserves-to-imports ratio, and conducts a sensitivity analysis exercise for each group of countries. From now on, the constant-elasticity-of-substitution (CES) utility function is used:

$$u(c_{H,t}, c_{F,t}) = \frac{\sigma}{\sigma - 1}\left[\theta^{\frac{1}{\gamma}}(c_{H,t} - \underline{c}_H)^{1-\frac{1}{\gamma}} + (1-\theta)^{\frac{1}{\gamma}}(c_{F,t} - \underline{c}_F)^{1-\frac{1}{\gamma}}\right]^{\frac{1-1/\sigma}{1-1/\gamma}}, \qquad (2)$$

with $\gamma > 0$ the elasticity of substitution between home and foreign goods, θ the preference for home goods, $1/\sigma > 0$ the coefficient of relative risk aversion, and $\underline{c}_H$ and $\underline{c}_F$ the subsistence consumption level of home and foreign goods. Note that when $\gamma = \sigma = 1$, this utility function reduces to the Cobb-Douglas utility used in the previous section. In addition, a Stone-Geary preference specification will be useful when calibrating the model to sub-Saharan African countries whose consumption is close to subsistence levels.[17]

In order to capture the urgency posed by some disasters, a monthly frequency is used for the calibration. Indeed, the main disruptions caused by a natural disaster such as a hurricane do not happen over the course of a year but over a few weeks or months. Hurricanes are sudden and short-lived events and the shortage of foreign exchange may materialize in the first weeks after the shock, not the next quarter or year. As a result, imports may drop to close to zero in the immediate aftermath of the shock with an arbitrarily large utility loss if a country does not hold any international reserves. A yearly frequency would smooth out the import loss and mask the utility loss given the concavity of the utility function.

[16] Again, a higher level of aid in the immediate aftermath of a disaster would lower the optimal reserve level even further. However, as argued in a previous footnote, a rapid response on a large scale is unlikely in the first months after the shock.

[17] This specification is consistent with the evidence from Ogaki and Zhang (2001) and Ogaki, Ostry, and Reinhart (1996) that the relative risk aversion coefficient is a decreasing function of wealth in poor countries.

Using the evidence from Section I on the impact of disasters and terms-of-trade shocks, the parameters of the model can be calibrated and the optimal level of international reserves estimated for countries from the Caribbean and the Sahel region. Table 1 presents the calibration parameters used for each country group. The monthly probability of shocks is set to match the estimates from Section I, and the output and export loss parameters are fixed to match the empirical ones. The size of the export sector is chosen to match the average exports-to-GDP ratio of the group, and the preference for foreign goods is set accordingly. The monthly discount factor β is set to 0.9966 and the coefficient of risk aversion to 5. Absent strong evidence regarding the elasticity of substitution between home and foreign goods, a value of 0.3 is used, as in Agenor, Bayraktar, and Aynaoui (2008) for the case of low-income countries. To calibrate the opportunity cost of holding reserves, one of Jeanne's (2007) suggestions is followed and the difference between higher-yielding domestic investment opportunities and the return on 10-year U.S. treasury bonds is considered. Caselli and Feyrer (2007) compute the return to capital in a sample of high- and low-income economies and document an average annual real return close to 7 percent in low-income countries. Because 10-year treasury bonds averaged a real rate of return of roughly 3.5 percent over 1963–2007, this approach leads to an opportunity cost of reserves of roughly 3.5 percent a year. Finally, unless otherwise noted, the subsistence levels of consumption are fixed to zero.

Finally, to numerically solve the Bellman equation $V(R_t)$, value function iteration on a grid is used for reserves holding spanning zero to five months of imports.[18]

Self-Insurance against Natural Disasters

Hurricanes in the Caribbean

In a "normal" state, the average Caribbean country exports and imports, respectively, 30 and 40 percent of its output. "Transfers" provide the remaining 10 percent of the financing. Consistent with Section I, a major hurricane hits every 25 years. Given that hurricanes are sudden events with little persistence and maximum disruption in impact, it is assumed here that a natural disaster brings exports to a full stop for some time. To estimate that time, the number of months with zero imports necessary to match the total exports loss of 10 percent identified in Section I is calculated. That way, one can estimate that a hurricane disrupts exports for an average of one month and a quarter while output drops by 36 percent.[19] Given that countries from

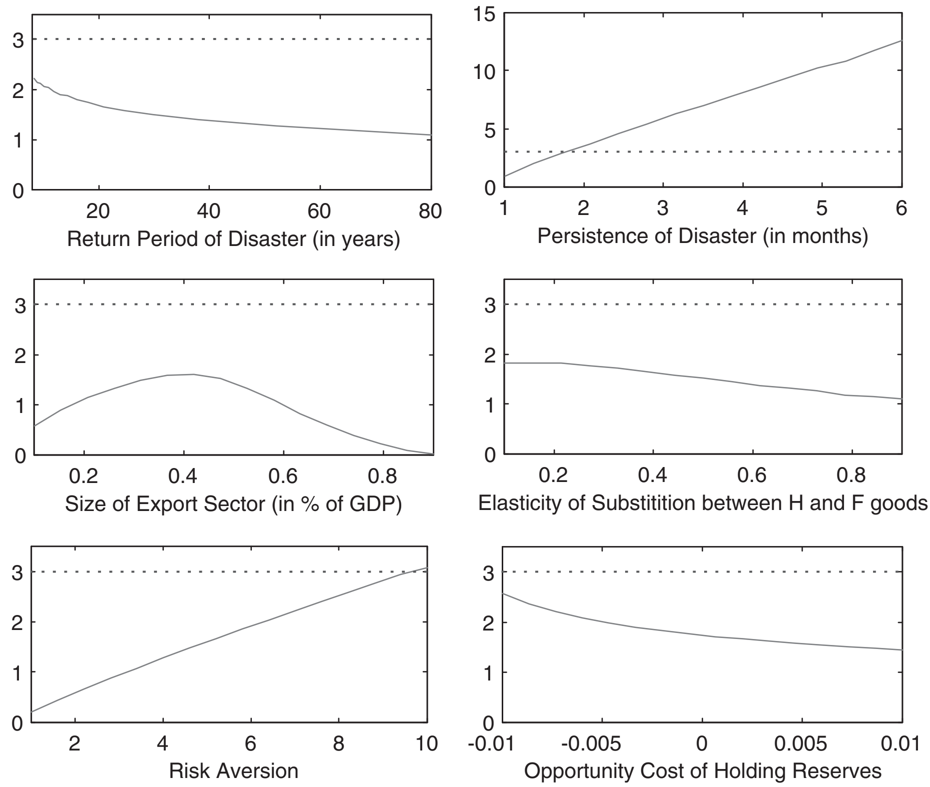
[18]For each country, I choose a grid size large enough (150 grid points) to ensure that the optimal reserves level (rounded at the second decimal) does not change with an increase in the grid's density. I start with an initial guess $V_0 = 0$ and stopping criterion $e = 10^{-5}$.

[19]A more realistic assumption would be to assume a gradual recovery phase starting one month after the shock. The present calibration exercise is mostly illustrative but could be easily extended to a richer setting.

Table 1. Calibration (*Monthly frequency*)

		Caribbean		Sahel	
		Hurricanes	Terms-of-trade shocks	Droughts	Terms-of-trade shocks
Probability of disaster (Return period)	π^{nd}	0.33% (25 years)	0.55% (17 years)	0.66% (12 years)	0.88% (10 years)
Persistence of disaster (Duration)	π^{dn}	83% (1.25 months)	8% (1 year)	16% (6 months)	8% (1 year)
Output loss	$1-\eta_Y$	36%	0	1%	0
Exports loss	$1-\eta_X$	100% (10% yearly)	0	16% (8% yearly)	0
Terms-of-trade loss	$1-\eta_\varepsilon$	0	10%	0	15%
Exports-to-GDP ratio	δ	0.4	0.4	0.2	0.2
Risk aversion	$—^{-1}$	5	5	5	5
Elasticity of substitution	$\circ$	0.3	0.3	0.3	0.3
Growth rate	g	0.20%	0.20%	0%	0%
Subsistence level of imports (percent of initial GDP)	c_F	0%	0%	26%	26%
Transfers (percent of initial GDP)	Tr	10%	10%	10%	10%
Optimal reserves-to-imports ratio (in months of imports)	R^*/c_F	1.42	<0.01	1.93	2.43
Shocks combined		1.52		4.10	

Figure 5. Optimal Reserves-to-Import Ratio (in months of imports) in the Caribbean: Sensitivity Analysis



the Caribbean have fixed exchange-rate regimes in the majority of cases, the terms of trade is kept constant during disasters. Finally, GDP per capita in the Caribbean grew at an average rate of 2.5 percent per year over the past 40 years so the monthly growth rate *g* was set to 0.20.

The estimated optimal level of reserves covers 1.42 months of imports, that is, slightly more than the expected duration of the disaster. Figure 5 illustrates the sensitivity of the results to key parameters: the return period of disasters (that is, the disaster's probability), the disaster's persistence, the size of the export sector, the opportunity cost of holding reserves, the subsistence level of imports, and the coefficient of risk aversion. In each case, the analysis here starts from the baseline calibration and vary one parameter at a time to draw a number of conclusions. First, the optimal reserves-to-imports ratio increases with the shocks' probability (or return period) and the shock's persistence. Second, as we previously observed in the simpler case with log-utility, there is an inverse U-shape relationship between the size of the export sector and optimal reserves due to the interaction of two factors: the utility cost of accumulating reserves and the opportunity cost of maintaining a given level of reserves. Third, as the elasticity of substitution between domestically produced goods and imported goods increases from 0.1 to 0.9, the optimal reserves-to-import ratio decreases by about a month. Fourth, looking at a range of plausible values of risk aversion, optimal reserves reach three months of imports only for large degrees of risk aversion. Finally, given the wide range of opportunity costs spanned by the simulation (from –10 to $+10$ percent per year), the optimal R^{*} varies comparatively little. This happens because the import sector represents only 40 percent of GDP in the baseline calibration. The total cost of holding reserves rR^{*} is not large and has thus only a small impact on the level of reserves.[20]

Droughts in the Sahel

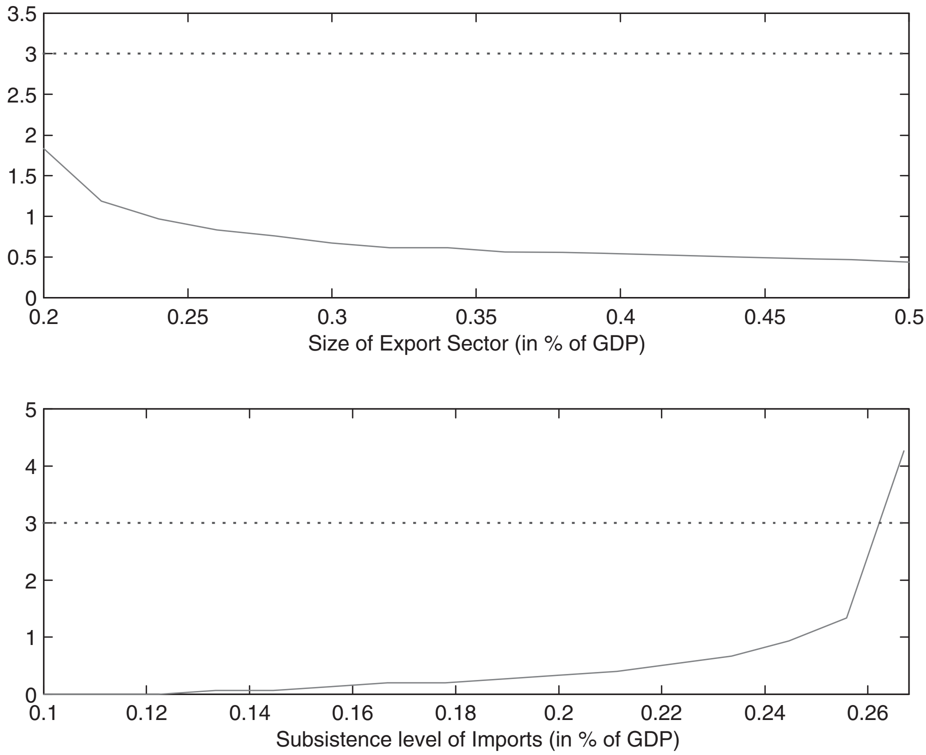
The average country from the Sahel region faces one major drought every 12 years. Droughts are more frequent than hurricanes and cause little physical damage but are also more persistent. Accordingly, it is assumed that annual output growth is unaffected while exports drop by 16 percent for six months, consistent with the 8 percent annual decline documented in Section I. The average Sahel country exports and imports, respectively, 20 and 30 percent of its output. Transfers provide the remaining 10 percent of the financing. Because most countries from the Sahel are part of the CFA Franc zone, the terms of trade is kept constant during disasters. Finally, to capture the situation in sub-Saharan Africa, where most of the population lives close to subsistence levels and where a small drop in consumption can have disastrous consequences, I postulate a nonzero subsistence level of imports.[21] Indeed, the 2008 riots in sub-Saharan Africa following the price increase of a number of food products shows that consumption in normal times is very close to subsistence levels. To calibrate $\underline{c}_F$, it is assumed that the 2008 riots were the result of consumption reaching subsistence levels. Given that the riots were triggered by an increase in food prices of 50 percent in one year and that the food basket represents roughly 40 percent of imports for an average Sahel country, the subsistence level of imports is set at 20 percent of "normal" imports, that is, 26 percent of "normal" GDP. However, the sensitivity of the results to a range of subsistence levels is also presented. Finally the GDP per capita growth rate in sub-Saharan Africa was zero or negative over the past 40 years so the monthly growth rate g is set to 0.

The estimated optimal reserves-to-import ratio is 1.93 months. Although the shock is less violent than with hurricanes, its duration makes it costly as it brings the population close to subsistence levels (that is, close to famine levels) for a long time. This provides a strong rational for holding

[20]In countries with a larger import sector (for example, 80 percent of GDP), simulations show that the opportunity cost of holding reserves plays a much more important role.

[21]Given the small effect of a drought on exports, the optimal reserves-to-import ratio is close to zero at 0.08 months if the subsistence level of imports is set to zero.

Figure 6. Optimal Reserves-to-Import Ratio (in months of imports) in the Sahel: Sensitivity Analysis



international reserves. When a drought occurs, reserves are used progressively to minimize the decline in imports over the expected duration of the drought. Figure 6 presents the sensitivity analysis and gives similar conclusions to the ones drawn for the Caribbean. Although the optimal reserves-to-import ratio depends on a country's characteristics, its level is always below three months except when parameters such as the subsistence level of imports, risk aversion, or the drought's persistence take very high values.

Self-Insurance against Terms-of-Trade Shocks

This subsection considers the impact of terms-of-trade shocks on the optimal level of international reserves. Caribbean economies tend to be less concentrated than in the Sahel, and the primary sector represents a smaller share of GDP. As a result, Caribbean countries are less sensitive to fluctuations in prices of raw materials, agricultural products, and staples. Using data on major terms-of-trade shocks since 1960, I find that while an average Caribbean country faces a 10 percent decline in its terms of trade every 17 years, an average Sahel country faces a 15 percent decline every 10 years.[22]

By calibrating the model to these transition probabilities and assuming that a terms-of-trade shock lasts for a year with no other economic impact on output and exports than the depreciation in ε, I find that the optimal reserves level represents less than 0.01 months of import for a Caribbean country but 2.43 months for a Sahel country.[23] Again, by bringing consumption of foreign goods close to subsistence levels for a long time, terms-of-trade shocks provide a strong rational for holding international reserves in the Sahel. In the Caribbean, however, a terms-of-trade shock lowers imports from 50 percent of GDP to slightly less than 45 percent, a small welfare loss given the concavity of the utility function.

Self-Insurance against Natural Disasters and Terms-of-Trade Shocks

This subsection estimates the optimal reserves-to-import ratio for each region by taking into account the possibility of natural disasters *and* terms-of-trade shocks. To do so, the two-state Markov process from Section III is generalized to three states. Home can either be in a "normal" state, facing a terms-of-trade shock, or facing a natural disaster, and the transition probabilities are the ones used previously.

The reserves target represents 1.52 months of import for a Caribbean country, only slightly more than that described earlier as self-insurance against natural disasters is the main motive for holding international reserves. For a country from the Sahel, however, the optimal reserves level stands at 4.10 months of imports, as droughts and terms-of-trade shocks are equally important disturbances.

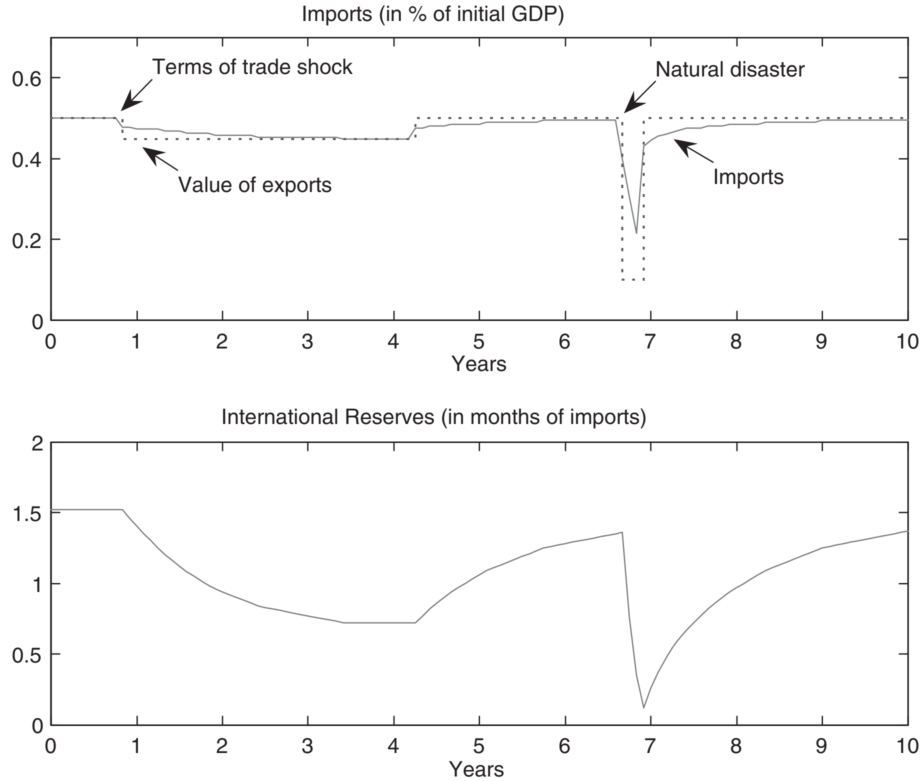
Figure 7 shows the impact of a terms-of-trade shock and a natural disaster on exports, imports, and international reserves in the Caribbean. Although Home initially keeps imports close to normal after the terms-of-trade shock, it progressively slows down the use of its reserves and reduces imports to avoid using too much of foreign exchange. Because the shock lasts much longer than expected (3.5 years instead of 1 year), Home stops using reserves after some time so as to keep enough reserves to respond to a hurricane. After the hurricane, exports drop by 50 percentage points, but imports decline by only 10 percentage points on impact thanks to the quick use of international reserves.

Figure 8 simulates the evolution of international reserves in a Sahel country. although Caribbean countries face the possibility of large, but rare,

[22]A worsening in the terms of trade is considered to be major when it is above 10 percent, based on data drawn from the IMF's *World Economic Outlook*. Terms-of-trade shocks that coincide with natural disasters are ignored.

[23]These assumptions are consistent with estimates of the economic impact of a terms-of-trade shock in each region. The empirical analysis (similar to the one conducted in Section I) is available upon request.

Figure 7. Impact of External Shocks in the Caribbean

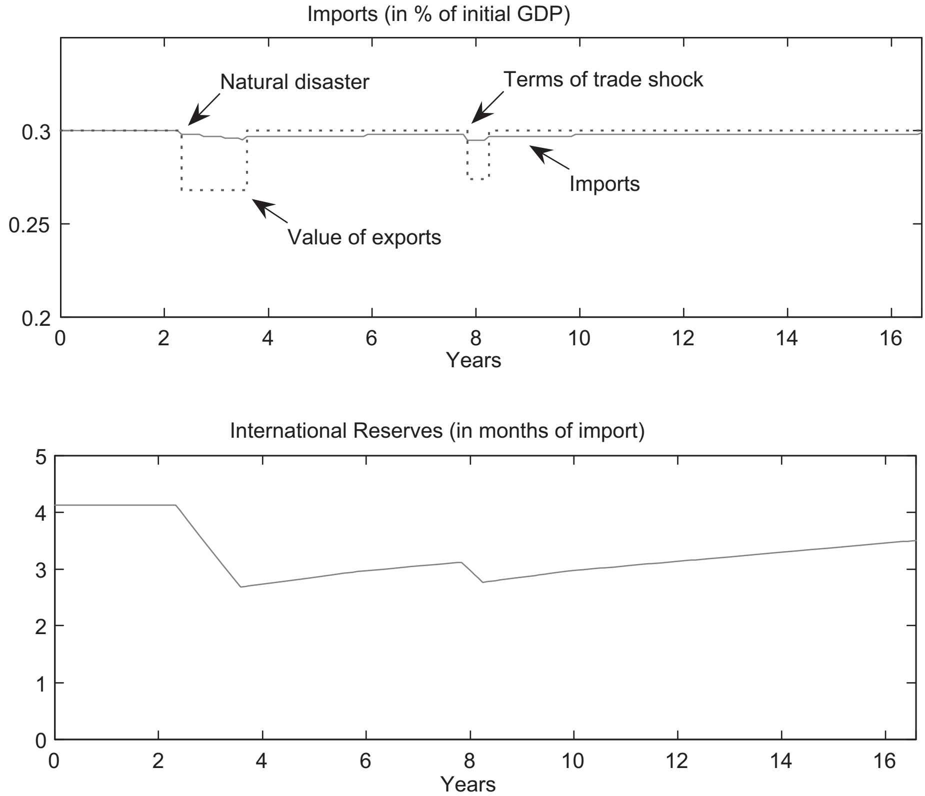


disruptive shocks, droughts and terms-of-trade shocks induce only mild, albeit frequent, declines in foreign exchange inflows. When hit by a terms-of-trade shock, Caribbean countries still face the possibility of very disruptive shocks and cannot afford to use too much reserves. This is not the case in the Sahel however, and in Figure 8 Home does not progressively slow down the use of its reserves. Similarly while Caribbean countries have to accumulate reserves at a fast pace, Sahel countries can smooth reserves accumulation.

Policy Implications

The main conclusion of this exercise is that the optimal reserves level is very sensitive to the parameters calibration. As a result, rules of thumb such as maintaining reserves equivalent to three months of import give only imprecise benchmarks. Although an average Caribbean country only needs one-and-a-half months of imports, an average Sahel country needs over four months. First, small parameter changes can have large consequences on the optimal reserves level. For example, depending on the size of the export

Figure 8. Impact of External Shocks in the Sahel



sector, the optimal reserves level in the Caribbean can take values between zero and two months of imports in the baseline calibration. Similarly, a shock's persistence of two months calls for roughly three months of imports in reserves but a persistence of three months already calls for five months.[24] Second, while some parameters play a critical role in a country, they can be almost irrelevant in another one. For example, the opportunity cost of holding reserves is negligible in a country with a small sector but it becomes determinant when exports represent a large share of GDP.

Finally, note that the average reserves level over time can be very different from the optimal level in steady-state, and one cannot evaluate a country's target by simply looking at its historical average. For example, the optimal reserves-to-import ratio is above four months for countries from the Sahel but the average reserves level is only at 3.25 months over the 16 years of the simulation.

[24]Note that the persistence of the shock can also be interpreted as the time taken by the international community to intervene and provide assistance that compensates for the loss in foreign exchange inflows.

IV. Conclusion

This paper develops an analytical framework that helps to quantify the level of reserves that can be rationalized in terms of insurance against large external shocks, such as natural disasters and terms-of-trade shocks. By calibrating the model, it is estimated that the optimal amount of international reserves for two groups of countries hit by different natural disasters: the Caribbean, hit by hurricanes, and the Sahel, hit by drought.

The calibration exercise shows that the optimal reserves level can be very sensitive to the parameters calibration, and the model needs to be carefully calibrated to evaluate each country's needs. As a result, rules of thumb such as maintaining reserves equivalent to three months of imports can only give imprecise benchmarks. Although an average Caribbean country only needs one-and-a-half months of imports, an average Sahel country needs over four months. Indeed, small changes in key parameters such as the size and persistence of shocks hitting a country, the importance of the export sector, or the degree of risk aversion, can have large consequences on the optimal reserves level.

An interesting extension would be to use a similar framework to evaluate the optimal size of sovereign wealth funds for economies relying mostly on primary commodities. Although the income provided by natural resources can provide large foreign exchange inflows, price volatility as well as uncertainty about the exact amount of natural resources available call for the accumulation of reserves to smooth price fluctuations and to provide an alternative source of revenue.

APPENDIX

Approximated Closed-Form Solution for the Reserves-to-Import Ratio

One starts by writing-up the agent's first-order condition:

$$\frac{1-\theta}{c_F}=\frac{\beta}{1+g}(1-\pi^{nd})\left(\frac{1-\theta}{c_F'^{n}}-\frac{r\theta}{c_H'^{n}}\right)+\beta\pi^{nd}\left(\frac{1-\theta}{c_F'^{d}}-\frac{r\theta}{c_H'^{d}}\right)$$

or

$$\frac{1-\theta}{\delta\varepsilon-gR}=\frac{\beta}{1+g}(1-\pi^{nd})\left(\frac{1-\theta}{\delta\varepsilon-gR}-\frac{r\theta/\varepsilon}{1-\delta-rR^{*}/\varepsilon}\right)$$
$$+\frac{\beta}{1+g}\pi^{nd}\left(\frac{1-\theta}{\varepsilon\delta\eta_{\varepsilon}\eta_{X}\eta_{Y}+R^{*}-(1+g)R'}-\frac{r\theta/\eta_{\varepsilon}\varepsilon}{(1-\delta\eta_{\delta})\eta_{Y}-rR^{*}/\eta_{\varepsilon}\varepsilon}\right),$$

with R^{*} the steady-state optimal reserves to output ratio and R' the level of reserves one period after the shock.

This expression can be simplified by noting that $r\ll 1$, $R\ll 1$, $g\ll 1$ and $\pi^{nd}\ll 1$. Indeed, the opportunity cost of reserves will typically be smaller than 10 percent, and developing countries rarely have a reserves-to-output ratio in excess of 10 percent. An annual growth rate in the order of 4 percent translates into a monthly g of only 0.33 percent. In addition, we saw in Section I that disasters are extremely rare events, occurring less than once every

Table A1. List of Countries in Each Group

Caribbean	Sahel
Antigua and Barbuda	Benin
The Bahamas	Burkina Faso
Barbados	Cape Verde
Belize	Côte d'Ivoire
Dominica	The Gambia
Dominican Republic	Guinea
Grenada	Guinea Bissau
Haiti	Mali
Honduras	Mauritania
Jamaica	Niger
St. Kitts and Nevis	Senegal
St. Lucia	Sierra Leone
St. Vincent and the Grenadines	Togo
Trinidad and Tobago	

10 years, so that π^{nd} is less than 1 percent. Hence, to a first order, one can approximate $1-\theta/\delta\varepsilon-gR$ with $1-\theta/\delta\varepsilon$, because gR is second order, and one can approximate $r\theta/1-\delta-rR^*$ with $r\theta/1-\delta$, because rR is second order, and given the parameters' calibration for η_Y and η_ε (see Table 1), the fourth term on the right-hand side is of second order ($\pi^{nd}r$ is second order) so that the first-order condition can be rewritten as

$$\frac{1-\theta}{\delta\varepsilon}\left(1-\frac{\beta(1-\pi^{nd})}{1+g}\right)\approx-\frac{\beta}{1+g}(1-\pi^{nd})\frac{r\theta/\varepsilon}{1-\delta}+\frac{\beta}{1+g}\pi^{nd}\frac{1-\theta}{\varepsilon\delta\eta_\varepsilon\eta_X\eta_Y+R^*-(1+g)R'}.$$

Rearranging, one has

$$\varepsilon\delta\eta_\varepsilon\eta_X\eta_Y+R^*-(1+g)R'\approx\frac{\frac{\beta}{1+g}\pi^{nd}(1-\theta)}{\frac{1-\theta}{\delta\varepsilon}\left(1-\frac{\beta(1-\pi^{nd})}{1+g}\right)+\frac{\beta}{1+g}(1-\pi^{nd})\frac{r\theta/\varepsilon}{1-\delta}},$$

so that

$$R^*-(1+g)R'\approx\frac{\pi^{nd}}{\frac{1}{\delta\varepsilon}\left(\frac{1+g}{\beta}-(1-\pi^{nd})\right)+(1-\pi^{nd})\frac{r\theta/\varepsilon}{(1-\delta)(1-\theta)}}-\varepsilon\delta\eta_\varepsilon\eta_X\eta_Y.$$

Assuming that the agent uses almost all of its reserves at once so that $R'\ll R^*$, one obtains the expression for the steady-state reserves-to-output ratio:

$$R^*\approx\frac{\pi^{nd}}{\frac{1}{\delta\varepsilon}\left(\frac{1+g}{\beta}-(1-\pi^{nd})\right)+(1-\pi^{nd})\frac{r\theta/\varepsilon}{(1-\delta)(1-\theta)}}-\varepsilon\delta\eta_\varepsilon\eta_X\eta_Y,$$

and the reserves-to-import ratio

$$\frac{R^*}{c_F}\approx\left[\frac{\pi^{nd}}{\left(\frac{1+g}{\beta}-(1-\pi^{nd})\right)+(1-\pi^{nd})\delta\frac{r\theta}{(1-\delta)(1-\theta)}}-\eta_\varepsilon\eta_X\eta_Y\right]\frac{1}{1+\frac{Tr}{\varepsilon\delta}}.$$

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